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Intrinsic Regularization Method in QCD [1]

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Abstract

There exist certain intrinsic relations between the ultraviolet divergent graphs and the convergent ones at the same loop order in renormalizable quantum field theories. Whereupon we may establish a new method, the intrinsic regularization method, to regularize those divergent graphs. In this paper, we apply this method to QCD at the one loop order. It turns out to be satisfactory: The gauge invariance is preserved manifestly and the results are the same as those derived by means of other regularization methods.

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1 Introduction

Over the decades, as is well known, a wide variety of regularization schemes have been developed in quantum field theory [1]. However, as every schemes heve their own distinct advantages and disadvantages, this topic is still one of the important and fundamental issues under investigation. One of the most challenging problem is perhaps how to preserve all properties of the original action manifestly and consistently.

A few years ago, a new regularization method named intrinsic vertex regularization was first proposed for the ϕ^4 theory by Wang and Guo [2]. The key point of the method is, in fact, based upon the following simple observation: For a given ultraviolet divergent function at certain loop order in a renormalizable QFT, there always exists a set of convergent functions at the same loop order such that their Feynman graphs share the same loop skeleton and the main difference is that the convergent ones have additional vertices of certain kind and the original one is the case without these vertices. This is, in fact, a certain intrinsic relation between the original ultraviolet divergent graph and the convergent ones in the QFT. It is this relation that indicates it is possible to introduce the regularized function for the divergent function with the help of those convergent ones so that the potentially divergent integral of the graph can be rendered finite while for the limiting case of the number of the additional vertices $q \to 0$ the divergence again becomes manifest in pole(s) of q.

To be concrete, let us consider a 1PI graph with I internal lines at one loop order in the ϕ^4 theory. Its superficial degree of divergences in the momentum space is

$$\delta = 4 - 2I.$$

When $I = 1$ or 2, the graph is divergent. Obviously, there exists such kind of graphs that they have additional q four-ϕ-vertices in the internal lines. Then the number of internal lines in these graphs is $I + q$ so that the divergent degree of the new 1PI graphs become

$$\delta' = 4 - 2(I + q).$$

If q is large enough, the new ones are convergent and the original divergent one is the case of $q = 0$. Thus, a certain intrinsic relation has been reached between the original divergent 1PI graph and the new convergent ones at the same loop order.

However, application of this method to QED runs into a difficulty. The problem is that, unlike the the ϕ^4 theory, the electron-photon vertex in QED carries a γ-matrix and is a Lorentz vector. As a result, simply inserting the vertex would increase the rank of the function as Lorentz tensors and would make the problem quite complicated. In order to overcome this difficulty, in [3, 4] the

authors introduced an alternative method. We follow the example of the ϕ^4 theory to demonstrate the key point of this method: One shifts the mass term of the ϕ field from m^2 to $m^2+\mu^2$ and regards $\mu^2\phi^2$ as a new vertex in addition to the vertex $\lambda\phi^4$. By inserting the new vertex into the internal ϕ lines in the graph of a given 1PI n-point divergent function, a set of new convergent functions can be obtained provided the number of inserted vertices, q, is large enough. Then one can introduce a new convergent function, the regularized function, and the potential infinity in the original 1PI n-point function may be recovered as the $q \to 0$ limiting case of that function. Obviously, the mass shifting method can be easily generalized to QED by simply shifting the electron mass from m to $m + \mu$ and regarding the term $-\mu\bar{\psi}\psi$ as a new vertex. In fact, as has been shown in [4], it turns out to be successful to QED. Nevertheless, it is not really intrinsic since the Feynman rules of the theory have to be modified. As a result, it may not completely work for non-Abelian gauge theories, *e.g.*, QCD, because generally it is not clear whether the gauge symmetry can be preserved manifestly for these theories, although such a proof for QED at one loop level has been given [6].

Very recently, we presented an improved approach in [7] to reexamine the ϕ^4 theory and QED, in which a new concept, *inserter*, was introduced. An inserter is a vertex or a pair of vertices linked by an internal line, in which the momenta of the external legs are all set to zero, and, if there are any, all the Lorentz indices are contracted in pair by the spacetime metric and all the internal gauge symmetry indices are contracted by the Killing-Cartan metric in the corresponding representation, so that as a whole an inserter always carries the vacuum quantum numbers, i.e. zero momentum, scalar in the spacetime symmetry, and singlet in internal and gauge symmetries. It is not hard to see that in any given QFT as long as a suitable kind of inserters are constructed with the help of the Feynman rules of the theory, some intrinsic relations between the divergent functions and convergent ones at the same loop order will be found by simply regarding the convergent ones as the ones given by suitably

inserting q-inserters in all internal lines in the given divergent ones. The crucial point of this approach, therefore, is very simple but fundamental, that is, the entire procedure is intrinsic in the QFT. There is nothing changed, the action, the Feynman rules, the spacetime dimensions etc. are all the same as that in the given QFT. This is a very important property which should shed light on the challenging problem mentioned at the beginning of the paper. Consequently, in applying to other cases all symmetries and topological properties there should be preserved in principle.

In what follows, we concentrate on how to apply the inserter approach to QCD at one loop order. We present the main steps and the results of the inserter regularization procedure for it. We find that, as is expected, the gauge invariance is preserved manifestly, and all results are the same as those derived by means of other regularization methods.

2 Intrinsic Regularization in QCD

The QCD Lagrangian, including ghost fields and gauge fixing terms, can be written as

$$\begin{aligned}\mathcal{L}_{QCD} &= -\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_c f^{abc} A^b_\mu A^c_\nu)(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} + g_c f^{ade} A^{d\mu} A^{e\nu}) \\ &\quad -\frac{1}{2\xi}(\partial_\mu A^{a\mu})^2 - \bar{\eta}^a \partial_\mu (\partial^\mu \delta^{ac} - g_c f^{abc} A^{b\mu})\eta^c + \bar{\psi}[i\gamma_\mu(\partial^\mu - i g_c A^{a\mu}\frac{\lambda^a}{2}) - m]\psi \ , \quad (1)\end{aligned}$$

and the Feynman rules are well known.

The main steps of the inserter approach for QCD may be stated more concretely as follows. First, we should construct the inserters in QCD. This work, with regards to simplicity and consistency with other theories, *e.g.*, the electroweak theory, may actually be done within a more general framework, namely, within the framework of the standard model in which QCD is contained. The explicit expressions of all inserters in the standard model have been preestablished in [7]. Here we merely list those relevant to QCD:

- The gluon-inserter:

$$I^{\{g\}ab}_{\mu\nu}(p) = -6 i g_c^2 C_2(\mathbf{8}) g_{\mu\nu}\delta^{ab}. \quad (2)$$

- The ghost-inserter:

$$I^{\{gh\}}_{a_1 a_2}(p) = -i g_c^2 C_2(\mathbf{8})\delta_{a_1 a_2}. \quad (3)$$

- The quark-inserter:

$$I^{\{q\}}(p) = -i\lambda_q. \quad (4)$$

In eqs.(2) and (3), $C_2(\mathbf{8})$ is the second Casimir operator valued in the adjoint representation of $SU_c(3)$ algebra. In eq.(4), λ_q takes value $\frac{g}{2}\frac{m_q}{M_W}$ in the standard model, but here its value is irrelevant for our purpose. It should be mentioned that here the quark inserters are constructed by borrowing the fermion-Higgs-vertex of Yukawa type from the standard model, this is in analogy with as occurs in QED. The issue has been discussed in detail in [7].

For a given divergent 1PI amplitude $\Gamma^{(n_f,n_g)}(p_1,\cdots,p_{n_f};k_1,\cdots,k_{n_g})$ at the one loop order with n_f external fermion lines and n_g external photon lines, we consider a set of 1PI amplitudes $\Gamma^{(n_f,n_g)}(p_1,\cdots,p_{n_f};k_1,\cdots,k_{n_g};q)$ which correspond to the graphs with, if the loop contained in the graph purely consists of fermion lines, all possible $2q$ insertions of the fermion inserter in the

internal fermion lines, or in other cases, all possible q insertions of the corresponding inserter in the internal boson (ghost) lines in the original graph. The divergent degree therefore becomes:

$$\delta = 4 - I_f - 2I_g - 2q.$$

If q is large enough, $\Gamma^{(n_f,n_g)}(p_1, \cdots, p_{n_f}; k_1, \cdots, k_{n_g}; q)$ are convergent and the original divergent function is the case of $q = 0$. Thus we reach a relation between the given divergent 1PI function and a set of convergent 1PI functions at the one loop order. In fact, the function of inserting the inserter(s) into internal lines is simply to raise the power of the propagator of the lines and to decrease the degree of divergence of given graph.

In order to regularize the given divergent function with the help of this relation, we need to deal with those convergent functions on an equal footing and pay attention to their differences due to the insertions. To this end, we introduce a new function:

$$\begin{aligned} &\Gamma^{(n_f,n_g)}(p_1, \cdots, p_{n_f}; k_1, \cdots, k_{n_g};\ q;\ \mu) \\ &\quad = (-i\mu)^{2q}(-i\lambda)^{-2q}\tfrac{1}{N_q}\sum \Gamma^{(n_f,n_g)}(p_1, \cdots, p_{n_f}; k_1, \cdots, k_{n_g};\ q) \end{aligned} \tag{5}$$

where μ is an arbitrary reference mass parameter, the summation is taken over the entire set of such N_q inserted functions, and the factor $(-i\lambda)^{-2q}$ introduced here, in which λ stands for λ_q for fermion loop and for g_c for other cases, is to cancel the ones coming from the inserters. It is clear that this function is the arithmetical average of those convergent functions and has the same dimension in mass, the same order in coupling constant with the original divergent 1PI function. Then we evaluate it and analytically continue q from the integer to the complex number. Finally, the original 1PI function is recovered as its $q \to 0$ limiting case:

$$\Gamma^{(n_f,n_g)}(p_1, \cdots, p_{n_f}; k_1, \cdots, k_{n_g}) = \lim_{q\to 0} \Gamma^{(n_f,n_g)}(p_1, \cdots, p_{n_f}; k_1, \cdots, k_{n_g}; q; \mu), \tag{6}$$

and the original infinity appears as pole in q. Similarly, this procedure should work for the cases at the higher loop orders in principle.

The divergent 1PI graphs at the one loop order in QCD are as follows: the gluon self-energy $\Pi^{ab}_{\mu\nu}(k)$, the quark self-energy $\Sigma(p)$, the ghost self-energy $\tilde{\Pi}^{ab}(p)$, renormalized by Z_3, Z_3^F and $\tilde{Z}_3$, the three-gluon vertex $\Gamma^{abc}_{\mu\nu\lambda}(k_1,\ k_2)$, the four-gluon vertex $\Gamma^{abcd}_{\mu\nu\lambda\tau}(k_1,\ k_2,\ k_3)$, the quark-gluon vertex $\Gamma^a_\mu(p', p)$, and the ghost-gluon vertex $\tilde{\Gamma}^{abc}_\mu(p', p)$, with the renormalization constant Z_1, Z_4, Z_1^F, $\tilde{Z}_1$. In addition, there is a mass shift for the quark, which we shall ignore. All corresponding graphs are listed in figures 1-7. As numerious diagrams are concerned, evaluating them one by one in detail would be much lengthy and unnecessary. In the next section, we will evaluate in detail the gluon self-energy $\Pi^{ab}_{\mu\nu}(k)$ as a typical example to show the main step of the approach, paying special attentions to the gauge inva

riance of the function. Then in the subsequent section, we will directly give all the results corresponding to other involved diagrams to verify the Slavnov-Taylor identities at one loop order.

3 Regularization and Evaluation of the Gluon Self-Energy $\Pi^{ab}_{\mu\nu}(k)$

Before the detailed evaluations are presented, we should first refer to a special problem which arises in any massless theories, *i.e.*, the genuine infrared divergence in these theories. In the regularization schemes, this problem usually appears as the lack of consistent definitions of the regularized Feynman integrals for the ones which are both ultraviolet and infrared divergent. For instance, in the dimensional regularization scheme, let's consider the massive integral

$$\int \frac{d^{2\omega}l}{(2\pi)^{2\omega}} \frac{1}{(l^2+m^2)^n} = \frac{i\Gamma(n-\omega)}{(4\pi)^{\omega}\Gamma(n)(m^2)^{n-\omega}} \equiv I(m,\ \omega,\ n)\ , \quad (\ m^2 \neq 0\ ) \tag{7}$$

which converges for ω complex; the parameter n is arbitrary but fixed. We note first of all that the limit $\lim_{m^2\to 0} I(m,\ \omega,\ n)$ may or may not exist, depending on the relative magnitudes of n and ω. But even if it did exist, another problem could arise as we approach four-space (provided the original integral is infrared divergent to begin with), because in general

$$\lim_{\omega\to 2}[\lim_{m^2\to 0} I(m,\ \omega,\ n)] \neq \lim_{m^2\to 0}[\lim_{\omega\to 2} I(m,\ \omega,\ n)],$$

so that the massless integral $\lim_{\omega\to 2}\int \frac{d^{2\omega}l}{(2\pi)^{2\omega}}\frac{1}{(l^2)^n}$ can not be derived unambiguously from the massive integral (7). Furthermore, the trick of inserting a finite mass into the integral and then allowing it to approach to zero at the end of the calculation is, in general, not a satisfactory prescription yet, because it spoils the gauge symmetry in the original theory, provided such a symmetry existed in the first place. To avoid this difficulty, 't Hooft and Veltman naively comjectured that

$$\lim_{\omega\to 2}\int \frac{d^{2\omega}l}{(2\pi)^{2\omega}}\frac{1}{(l^2)^n} = 0\ , \quad for\ \omega,\ n\ complex. \tag{8}$$

It has been shown that no inconsistencies occur, *e.g.*, in the Slavnov-Taylor identities [10], due to the acceptance of the above conjecture.

In our application of the present approach to QCD, as we will see, the same problem occurs, *e.g.*, in calculating the tadpole diagram Fig.1d of the gluon self-energy. To solve this problem, we employ a conjecture analogous to 't Hooft and Veltman's:

$$\lim_{q\to 0}\int \frac{d^4 l}{(2\pi)^4}\frac{1}{[(k-l)^2]^{Aq}(l^2)^{Bq+n}} = 0, \quad for\ q,\ n,\ complex,\ A\geq 0,\ B\geq 0,\ A+B=1\ . \tag{9}$$

Likewise, we will see that no inconsistencies occur due to the acceptance of the eq.(9).

Now we turn to evaluate in detail the gluon self-energy $\Pi^{ab}_{\mu\nu}(k)$ shown in Fig.1. The diagrams contributing to $\Pi^{ab}_{\mu\nu}(k)$ are four in number, namely, the gluon loop diagram, the ghost loop diagram, the quark loop diagrams, and the gluon tadpole. The integral expressions of the regularized diagrams in the momentum space are given in the appendix (For simplicity, we take the Feynman gauge $\xi = 1.$).

First, we consider the gluon loop contribution. From (19a), a little bit of algebra yields

$$\Pi^{ab}_{(A)\mu\nu}(k;q;\mu) = -\frac{1}{2}g_c^2[-6C_2(\mathbf{8})\mu^2]^q\delta^{ab}I_1 ,$$

with

$$I_1 = \frac{1}{N_q}\sum_{i=0}^{q}\int\frac{d^4p}{(2\pi)^4}\frac{-2p^2g_{\mu\nu}-(5k^2-2p\cdot k)g_{\mu\nu}-10p_\mu p_\nu+2k_\mu k_\nu+5p_\mu k_\nu+5k_\mu p_\nu}{(p^2)^{i+1}[(p-k)^2]^{q-i+1}} .$$

In the present case, $N_q = q+1$. Note that because of eq.(9), the contribution of the first term in the numerator of the above equation actually vanishes, so it can be neglected. Using the Feynman parameterization, we get

$$\begin{aligned} I_1 &= \frac{1}{q+1}\sum_{i=0}^{q}\frac{\Gamma(q+2)}{\Gamma(i+1)\Gamma(q-i+1)}\int_0^1 dx x^{q-i}(1-x)^i \\ &\times\int\frac{d^4p}{(2\pi)^4}\frac{(2x-5)k^2g_{\mu\nu}-10p_\mu p_\nu-(10x^2-10x-2)k_\mu k_\nu}{[p^2+x(1-x)k^2]^{q+2}} , \end{aligned} \tag{10}$$

where we have made a momentun shift: $p\to p-kx$, and the linear terms in p in the numerator have been dropped since they do not contribute to the integral. Now the integration over p can be performed by using the following formulas:

$$\int\frac{d^4p}{(2\pi)^4}\frac{(p^2)^\beta}{(p^2+M^2)^A} = \frac{i}{(4\pi)^2}\frac{\Gamma(2+\beta)\Gamma(A-2-\beta)}{\Gamma(A)}(M^2)^{2+\beta-A}, \tag{11a}$$

$$\int\frac{d^4p}{(2\pi)^4}\frac{(p^2)^\beta p_\mu p_\nu}{(p^2+M^2)^A} = \frac{i}{(4\pi)^2}\frac{1}{4}g_{\mu\nu}\frac{\Gamma(3+\beta)\Gamma(A-3-\beta)}{\Gamma(A)}(M^2)^{3+\beta-A}, \tag{11b}$$

$$\begin{aligned}\int\frac{d^4p}{(2\pi)^4}\frac{(p^2)^\beta p_\mu p_\nu p_\rho p_\sigma}{(p^2+M^2)^A} &= \frac{i}{(4\pi)^2}\frac{1}{24}(g_{\mu\nu}g_{\rho\sigma}+g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho}) \\ &\times\frac{\Gamma(4+\beta)\Gamma(A-4-\beta)}{\Gamma(A)}(M^2)^{4+\beta-A}. \end{aligned}\tag{11c}$$

From (10), (11a), and (11b) we get:

$$\begin{aligned} I_1 &= \frac{i}{(4\pi)^2}\sum_{i=0}^{q}\frac{\Gamma(q-1)}{(q+1)\Gamma(i+1)\Gamma(q-i+1)}\int_0^1 dx x^{q-i}(1-x)^i \\ &\times\frac{(q-1)[(2x-5)k^2g_{\mu\nu}-(10x^2-10x-2)k_\mu k_\nu]-5x(1-x)k^2g_{\mu\nu}}{[x(1-x)k^2]^q} \\ &= \frac{i}{(4\pi)^2}\frac{\Gamma(q-1)}{\Gamma(q+2)}\int_0^1 dx\frac{[q(2x-5)+(5x^2-7x+5)]k^2g_{\mu\nu}-(q-1)(10x^2-10x-2)k_\mu k_\nu}{[x(1-x)k^2]^q} , \end{aligned}$$

where in the last step the summation over i has been performed with the help of the binomial theorem. This expression also makes sense when we make an analytical continuation of q from integer to complex number. When $q \to 0$, using the expansion

$$\frac{\Gamma(q-1)}{\Gamma(q+2)} = -\frac{1}{q} - q + O(q^2), \quad [x(1-x)k^2]^q = 1 + q\ln[x(1-x)k^2] + O(q^2)\ ,$$

and making a rescaling of the parameter $\mu^2 \to -6C_2(\mathbf{8})\mu^2$, we get:

$$\begin{aligned}\Pi_{(A)\mu\nu}^{ab}(k;q;\mu) &= \frac{ig_c^2 C_2(\mathbf{8})}{(4\pi)^2}\delta^{ab}[(\frac{19}{12}k^2 g_{\mu\nu} - \frac{11}{6}k_\mu k_\nu)\frac{1}{q} \\ &\quad -(\frac{19}{12}k^2 g_{\mu\nu} - \frac{11}{6}k_\mu k_\nu)\ln\Big(\frac{k^2}{\mu^2}\Big) + (\frac{47}{36}k^2 g_{\mu\nu} - \frac{14}{9}k_\mu k_\nu)]\ . \end{aligned} \tag{12}$$

Clearly $\Pi_{(A)\mu\nu}^{ab}(k;q;\mu)$ does not conserve current, this is due to our choice to use a covariant gauge (rather than, for example, an axial gauge). For the sake of this choice, we have to introduce spurious gluon polarization states. These spurious states must be removed by taking the ghost loop contribution into account. We could have computed with an axial or "ghost free" gauge, but it is usually much easier to use the simple Feynman gauge and add in the ghost contribution.

To compute ghost loop contribution (19b), we use the same Feynman parameterization arriving at

$$\Pi_{(B)\mu\nu}^{ab}(k;q;\mu) = -\, g_c^2 C_2(\mathbf{8})[C_2(\mathbf{8})\mu^2]^q \delta^{ab} I_2\ ,$$

with

$$I_2 = \int_0^1 dx \int \frac{d^4p}{(2\pi)^4}\frac{p_\mu p_\nu - x(1-x)k_\mu k_\nu}{[p^2 + x(1-x)k^2]^{q+2}}\ ,$$

where momentun shift: $p \to p - kx$ has been made, and the linear terms in p in the numerator have been dropped. After performing the integration over p and taking the limit $q \to 0$ subsequently, we obtain

$$\begin{aligned}\Pi_{(B)\mu\nu}^{ab}(k;q;\mu) &= \frac{ig_c^2 C_2(\mathbf{8})}{(4\pi)^2}\delta^{ab}[(\frac{1}{12}k^2 g_{\mu\nu} + \frac{1}{6}k_\mu k_\nu)\frac{1}{q} \\ &\quad -(\frac{1}{12}k^2 g_{\mu\nu} - \frac{1}{6}k_\mu k_\nu)\ln\Big(\frac{k^2}{\mu^2}\Big) + \frac{5}{36}k^2 g_{\mu\nu} + \frac{1}{9}k_\mu k_\nu)]\ . \end{aligned} \tag{13}$$

Again, we find that $\Pi_{(B)\mu\nu}^{ab}(k;q;\mu)$ also does not conserve current. However, the non-conserving term in (13) exactly cancels that in (12), and makes $\Pi_{(A)\mu\nu}^{ab}(k;q;\mu)+\Pi_{(B)\mu\nu}^{ab}(k;q;\mu)$ gauge invariant, namely,

$$k^\mu[\Pi_{(A)\mu\nu}^{ab}(k;q;\mu) + \Pi_{(B)\mu\nu}^{ab}(k;q;\mu)] = 0\ .$$

Next, the quark loop contribution $\Pi_{(C)\mu\nu}^{ab}(k;q;\mu)$ need not be recalculated since it is just $\Pi_{\mu\nu}^{QED}(k;q;\mu)$, which can be found in [6, 7], multiplied by a color factor $N_f Tr(\mathbf{T}^a\mathbf{T}^b)$:

$$\begin{aligned}\Pi_{(C)\mu\nu}^{ab}(k;q;\mu) &= N_f Tr(\mathbf{T}^a\mathbf{T}^b)\Pi_{\mu\nu}^{QED}(k;q;\mu) = \frac{4ig_c^2 N_f C_2(\mathbf{3})}{(4\pi)^2}\delta^{ab}(k_\mu k_\nu - k^2 g_{\mu\nu}) \\ &\quad \times[\frac{1}{3q} + \frac{2}{3} - 2\int_0^1 dxx(1-x)\ln\frac{k^2x(1-x)-m^2}{\mu^2} + \cdots] \ . \end{aligned} \tag{14}$$

Obviously, $\Pi_{(C)\mu\nu}^{ab}(k;q;\mu)$ itself is gauge invariant.

Finally, as in dimensional regularization scheme, from eq.(9) we know that the contribution of the gluon tadpole diagram vanishes:

$$\Pi_{(D)\mu\nu}^{ab}(k;q;\mu) = 0 \ . \tag{15}$$

From (12), (13), (14), and (15) we obtain the final expression of the regularized gluon self-energy $\Pi_{\mu\nu}^{ab}(k;q;\mu)$:

$$\begin{aligned}\Pi_{\mu\nu}^{ab}(k;q;\mu) &= \Pi_{(A)\mu\nu}^{ab}(k;q;\mu) + \Pi_{(B)\mu\nu}^{ab}(k;q;\mu) + \Pi_{(C)\mu\nu}^{ab}(k;q;\mu) \\ &= \frac{ig_c^2}{(4\pi)^2}\delta^{ab}(k^2 g_{\mu\nu} - k_\mu k_\nu)\{[\frac{5}{3}C_2(\mathbf{8}) - \frac{4}{3}N_f C_2(\mathbf{3})]\frac{1}{q} + \cdots\} \ , \end{aligned} \tag{16}$$

which is the same as that derived by means of other regularization methods and satisfies the gauge invariance condition $k^\mu \Pi_{\mu\nu}^{ab}(k;q;\mu) = 0$.

4 Renormalization Constants and Slavnov-Taylor Identities at one Loop Order

So far we have calculated the gluon self-energy $\Pi_{\mu\nu}^{ab}(k)$ in detail by means of the intrinsic regularization method at one loop level. As we have expected, the result turns out to be gauge invariant. However, this is not enough for us to say that the method preserves the gauge invariance of QCD. As a complement, we should further show that the Slavnov-Taylor identities between the renormalization constants hold, namely,

$$\frac{Z_1}{Z_3} = \frac{Z_1^F}{Z_3^F} = \frac{\tilde{Z}_1}{\tilde{Z}_3} = \frac{Z_4}{Z_1} \ . \tag{17}$$

It is these identities that guarantee that the renormalized theory possess the same gauge theory structure as the original one. Moreover, they are essential for proving the renormalizability and unitarity of the theory.

To verify these identities, all divergent 1PI graphs, not only $\Pi^{ab}_{\mu\nu}(k)$, but also others, must be taken into account. After lengthy and tedious calculations, we get

$$Z_1 = 1 + \frac{g_c^2}{(4\pi)^2}[\frac{2}{3}C_2(\mathbf{8}) - \frac{4}{3}N_f C_2(\mathbf{3})]\frac{1}{q} \tag{18a}$$

$$Z_3 = 1 + \frac{g_c^2}{(4\pi)^2}[\frac{5}{3}C_2(\mathbf{8}) - \frac{4}{3}N_f C_2(\mathbf{3})]\frac{1}{q} \tag{18b}$$

$$Z_4 = 1 + \frac{g_c^2}{(4\pi)^2}[-\frac{1}{3}C_2(\mathbf{8}) - \frac{4}{3}N_f C_2(\mathbf{3})]\frac{1}{q} \tag{18c}$$

$$Z_1^F = 1 - \frac{g_c^2}{(4\pi)^2}[C_2(\mathbf{8}) + \frac{8}{3}C_2(\mathbf{3})]\frac{1}{q} \tag{18d}$$

$$Z_3^F = 1 - \frac{g_c^2}{(4\pi)^2}\frac{8}{3}C_2(\mathbf{3})\frac{1}{q} \tag{18e}$$

$$\tilde{Z}_1 = 1 - \frac{g_c^2}{(4\pi)^2}\frac{C_2(\mathbf{8})}{2}\frac{1}{q} \tag{18f}$$

$$\tilde{Z}_3 = 1 + \frac{g_c^2}{(4\pi)^2}\frac{C_2(\mathbf{8})}{2}\frac{1}{q} \tag{18g}$$

It is easy to see that the Slavnov-Taylor identities (17) indeed holds. For a close observation, we note that although these expressions are calculated in a specific gauge therefore their gauge dependence are not explicit, they are in fact all gauge dependent (For instance, in the axial gauge, the effective Lagrantian has no ghosts and has the same structure as in QED, resulting in the identity $Z_1^F = Z_3^F$, which is patently untrue in the Feynman gauge.). We also remark that the fermion contribution to the vector boson quartic self-interaction must diverge if the Slavnov-Taylor identities (17) are to hold because we explicitly see that the ratio $Z_1^F/Z_3^F Z_3^{\frac{1}{2}}$, by which the bare coupling constant is related to the renormalized one, contains a fermion contribution. On the contrary, the corresponding box diagram of QED is finite. The reason is that in QED the box diagram's divergence vanishes only upon symmetrization of the external photon lines denoted only by their vector in

dices, while in QCD the symmetrization of the external lines can be performed in two ways: by symmetrizing on both vector and group indices, which as in QED gives no divergence, *or* by antisymmetrizing on both vector and group indices. It is this new contribution which deverges. The same reason can be applied to the fermion contribution to the triple gauge vertex.

5 Concluding Remarks

We have shown the main steps and results for the regularization of the divergent 1PI functions at the one loop order in QCD by means of the inserter proposal for the intrinsic regularization method. It turns out to be satisfactory: The gauge invariance is preserved manifestly and the results are the same as those derived by means of other regularization methods. It is natural to expect that this proposal should be available to the cases at higher loop orders in principle.

The renormalization of the QCD under consideration in this scheme should be the same as in usual approaches. Namely, we may subtract the divergent part of the n-point functions at each loop order by adding the relevant counterterms to the action. The renormalized n-point functions are then evaluated from the renormalized action. In the limiting case, we get the finite results for all correlation functions.

It should be mentioned that the method presented here is somewhat analogous to the analytic regularization method developed by Speer [11, 12]. However, the two methods are in fact different: As is well known, the analytic regularization method violates unitarity, this is due to the fact that it actually continuing the power of propagators in an arbitrary way. While in our method this not the case. Here we trie to find out a procedure of regularization from some physical principles. That is, giving a ultraviolet divergent process, one can always find a set of convergent function obtainable from existing Feymann rules and for the limiting case it turns to be the original ultra-divergent one. Or from another point of view, given a ultra-divergent function, one can always extract a set of convergent functions from a certain physical process, which is gauge invariant. After taking the limitation of the convergent functions, the divergent function is naturally regularized. There is nothing changed, the action, the Feynman rules, the spacetime dimensions etc. are all the same as that in the given QFT. From this viewpoint, one should have no doubt of gauge invariance and the unitarity of the method since the sum of the convergent functions comes from a certain physical process.

Application of our approach to gauge theories containing spontaneous symmetry breaking such as the standard model should be straightforward. Also, it will be much helpful to apply our approach to some other cases, such as anomalies, SUSY theories *etc.*, since in these cases the symmetries and topological properties are sensitive to the spacetime dimensions and the number of fermionic degrees of freedom *etc.*, thus we are unable to tackle them consistently by means of the hitherto well-known regularization methods such as dimensional regularization method. It is reasonable to expect that the approach presented here should be able to get rid of those problems. We will investigate these issues in detail elsewhere.

Appendix: Integral Expressions of the Divergent 1PI Graphs at One Loop Level in QCD

There are a number of divergent iPI graphs at one loop level in QCD, which contribute to the gluon self-energy $\Pi^{ab}_{\mu\nu}(k)$, the quark self-energy $\Sigma(p)$, the ghost self-energy $\tilde{\Pi}^{ab}(p)$, the three-gluon vertex $\Gamma^{abc}_{\mu\nu\lambda}(k_1,\ k_2)$, the four-gluon vertex $\Gamma^{abcd}_{\mu\nu\lambda\tau}(k_1,\ k_2,\ k_3)$, the quark-gluon vertex $\Gamma^a_\mu(p',p)$, and the ghost-gluon vertex $\tilde{\Gamma}^{abc}_\mu(p',p)$ respectively. In what follows we present the integral expressions of the regularized diagrams in the momentum space (in Feynman gauge $\xi = 1$).

1. The integral expressions of the regularized diagrams contributing to $\Pi^{ab}_{\mu\nu}(k)$:

$$\begin{aligned}
\Pi^{ab}_{(A)\mu\nu}(k;q;\mu) &= \frac{1}{2}g_c^{2-2q}\mu^{2q}[-6ig_c^2C_2(\mathbf{8})]^q Tr(\mathbf{F}^a\mathbf{F}^b)\frac{1}{N_q}\sum_{i=0}^{q}\int\frac{d^4p}{(2\pi)^4} \\
&\quad\times G_{\mu\rho_1\sigma_1}(k,-p,p-k)G_{\nu\rho_2\sigma_2}(-k,p,k-p) \\
&\quad\times g^{\rho_1\rho_2}g^{\sigma_1\sigma_2}\Big(\frac{-i}{p^2}\Big)^{i+1}\Big(\frac{-i}{(p-k)^2}\Big)^{q-i+1}\ ,
\end{aligned} \tag{19a}$$

$$\begin{aligned}
\Pi^{ab}_{(B)\mu\nu}(k;q;\mu) &= g_c^{2-2q}\mu^{2q}[-ig_c^2C_2(\mathbf{8})]^q Tr(\mathbf{F}^a\mathbf{F}^b)\frac{1}{N_q}\sum_{i=0}^{q}\int\frac{d^4p}{(2\pi)^4} \\
&\quad\times p_\mu(p-k)_\nu\Big(\frac{-i}{p^2}\Big)^{i+1}\Big(\frac{-i}{(p-k)^2}\Big)^{q-i+1}\ ,
\end{aligned} \tag{19b}$$

$$\begin{aligned}
\Pi^{ab}_{(C)\mu\nu}(k;q;\mu) &= g_c^2\mu^{2q}(-i\lambda_q)^{2q}N_f Tr(\mathbf{T}^a\mathbf{T}^b)\frac{1}{N_q}\sum_{i=0}^{2q}\int\frac{d^4p}{(2\pi)^4} \\
&\quad\times Tr[\gamma_\mu\Big(\frac{1}{\not p-\not k-m}\Big)^{i+1}\gamma_\nu\Big(\frac{1}{\not p-m}\Big)^{2q-i+1}]\ ,
\end{aligned} \tag{19c}$$

$$\begin{aligned}
\Pi^{ab}_{(D)\mu\nu}(k;q;\mu) &= -\frac{1}{2}ig_c^{2-2q}\mu^{2q}[-6ig_c^2C_2(\mathbf{8})]^q[f^{abe}f^{cde}(g_{\mu\sigma}g_{\nu\rho}-g_{\mu\rho}g_{\nu\sigma}) \\
&\quad+f^{ace}f^{dbe}(g_{\mu\rho}g_{\nu\sigma}-g_{\mu\nu}g_{\rho\sigma})+f^{ade}f^{bce}(g_{\mu\nu}g_{\rho\sigma}-g_{\mu\sigma}g_{\nu\rho})] \\
&\quad\times g^{\sigma\rho}\delta^{cd}\int\frac{d^4p}{(2\pi)^4}\Big(\frac{-i}{p^2}\Big)^{q+1}\ ,
\end{aligned} \tag{19d}$$

where N_f is the number of flavors, $\mathbf{T}^a$ are generators of $SU_c(3)$ in fundamental representation, f^{abc} denote the structure constants of $SU_c(3)$, $(\mathbf{F}^a)^{bc} = -if^{abc}$ are generators of $SU_c(3)$ in adjoint representation, and

$$G_{\mu\nu\lambda}(p_1,p_2,p_3) = (p_1-p_2)_\lambda g_{\mu\nu} + (p_2-p_3)_\mu g_{\nu\lambda} + (p_3-p_1)_\nu g_{\lambda\mu}$$

comes from the three-gluon vertex.

2. The integral expressions of the regularized diagrams contributing to $\Gamma^{abc}_{\mu\nu\lambda}(k_1,\ k_2)$:

$$
\begin{aligned}
\Gamma^{abc}_{(A)\mu\nu\lambda}(k_1,\ k_2;q;\mu) \ =\ & ig_c^{3-2q}\mu^{2q}[-6ig_c^2 C_2(\mathbf{8})]^q Tr(\mathbf{F}^a\mathbf{F}^b\mathbf{F}^c)\frac{1}{N_q}\sum_{i=0}^{q}\sum_{j=0}^{q-i}\int\frac{d^4p}{(2\pi)^4} \\
& \times g^{\rho_1\sigma_2}g^{\rho_2\sigma_3}g^{\rho_3\sigma_1}G_{\nu\rho_2\sigma_2}(k_2,-k_2-p,p)G_{\lambda\rho_3\sigma_3}(-k_1-k_2,k_1-p,k_2+p) \\
& \times G_{\mu\rho_1\sigma_1}(k_1,-p,p-k_1)\Big(\frac{-i}{p^2}\Big)^{i+1}\Big(\frac{-i}{(p+k_2)^2}\Big)^{j+1}\Big(\frac{-i}{(p-k_1)^2}\Big)^{q-i-j+1}\ ,
\end{aligned}
\tag{20a}
$$

$$
\begin{aligned}
\Gamma^{abc}_{(B)\mu\nu\lambda}(k_1,\ k_2;q;\mu) \ =\ & -\frac{1}{2}ig_c^{3-2q}\mu^{2q}[-6ig_c^2 C_2(\mathbf{8})]^q\frac{1}{N_q}\sum_{i=0}^{q}\int\frac{d^4p}{(2\pi)^4}f^{as_1t_1} \\
& G_{\mu\rho_1\sigma_1}(k_1,-p,p-k_1)[f^{bcr}f^{t_2s_2r}(g_{\nu\sigma_2}g_{\lambda\rho_2}-g_{\nu\rho_2}g_{\lambda\sigma_2}) \\
& +f^{bt_2r}f^{s_2cr}(g_{\nu\rho_2}g_{\lambda\sigma_2}-g_{\nu\lambda}g_{\rho_2\sigma_2})+f^{bs_2r}f^{ct_2r}(g_{\nu\lambda}g_{\rho_2\sigma_2}-g_{\nu\sigma_2}g_{\lambda\rho_2})] \\
& \times\delta^{s_1s_2}\delta^{t_1t_2}g^{\rho_1\rho_2}g^{\sigma_1\sigma_2}\Big(\frac{-i}{p^2}\Big)^{i+1}\Big(\frac{-i}{(p-k_1)^2}\Big)^{q-i+1} \\
& +\{(\mu,a,k_1)\leftrightarrow(\nu,b,k_2)\}+\{(\mu,a,k_1)\leftrightarrow(\lambda,c,k_3)\}\ ,
\end{aligned}
\tag{20b}
$$

$$
\begin{aligned}
\Gamma^{abc}_{(C)\mu\nu\lambda}(k_1,\ k_2;q;\mu) \ =\ & -ig_c^{3-2q}\mu^{2q}[-ig_c^2 C_2(\mathbf{8})]^q Tr(\mathbf{F}^a\mathbf{F}^b\mathbf{F}^c)\frac{1}{N_q}\sum_{i=0}^{q}\sum_{j=0}^{q-i}\int\frac{d^4p}{(2\pi)^4} \\
& \times p_\mu(p+k_2)_\nu(p-k_1)_\lambda\Big(\frac{i}{p^2}\Big)^{i+1}\Big(\frac{i}{(p+k_2)^2}\Big)^{j+1}\Big(\frac{i}{(p-k_1)^2}\Big)^{q-i-j+1} \\
& +\{(\nu,b,k_2)\leftrightarrow(\lambda,c,k_3)\}\ ,
\end{aligned}
\tag{20c}
$$

$$
\begin{aligned}
\Gamma^{abc}_{(D)\mu\nu\lambda}(k_1,\ k_2;q;\mu) \ =\ & ig_c^3\mu^{2q}(-i\lambda_q)^{2q}Tr(\mathbf{T}^a\mathbf{T}^b\mathbf{T}^c)\frac{1}{N_q}\sum_{i=0}^{2q}\sum_{j=0}^{2q-i}\int\frac{d^4p}{(2\pi)^4} \\
& \times Tr[\gamma_\mu\Big(\frac{i}{\not p-\not k_1-m}\Big)^{i+1}\gamma_\lambda\Big(\frac{i}{\not p+\not k_2-m}\Big)^{j+1}\gamma_\nu\Big(\frac{i}{\not p-m}\Big)^{2q-i-j+1}] \\
& +\{(\nu,b,k_2)\leftrightarrow(\lambda,c,k_3)\}\ ,
\end{aligned}
\tag{20d}
$$

3. The integral expressions of the regularized diagrams contributing to $\Gamma^{abcd}_{\mu\nu\lambda\tau}(k_1,\ k_2,\ k_3)$:

$$
\begin{aligned}
\Gamma^{abcd}_{(A)\mu\nu\lambda\tau}(k_1,\ k_2,\ k_3;q;\mu) \ =\ & g_c^{4-2q}\mu^{2q}[-6ig_c^2 C_2(\mathbf{8})]^q Tr(\mathbf{F}^a\mathbf{F}^b\mathbf{F}^c\mathbf{F}^d)\frac{1}{N_q}\sum_{i=0}^{q}\sum_{j=0}^{q-i}\sum_{l=0}^{q-i-j} \\
& \times\int\frac{d^4p}{(2\pi)^4}G_{\mu\rho_1\sigma_1}(k_1,-p,p-k_1)G_{\nu\rho_2\sigma_2}(k_2,-k_2-p,p) \\
& \times G_{\tau\rho_4\sigma_4}(-k_1-k_2-k_3,k_1-p,k_2+k_3+p) \\
& \times G_{\lambda\rho_3\sigma_3}(k_3,-k_2-k_3-p,k_2+p)g^{\rho_1\sigma_2}g^{\rho_2\sigma_3}g^{\rho_3\sigma_4}g^{\rho_4\sigma_1} \\
& \times\Big(\frac{-i}{p^2}\Big)^{i+1}\Big(\frac{-i}{(p+k_2)^2}\Big)^{j+1}\Big(\frac{-i}{(p+k_2+k_3)^2}\Big)^{l+1}\Big(\frac{-i}{(p-k_1)^2}\Big)^{q-i-j-l+1} \\
& +\{(\nu,b,k_2)\rightarrow(\lambda,c,k_3),\ (\lambda,c,k_3)\rightarrow(\tau,d,k_4),\ (\tau,d,k_4)\rightarrow(\nu,b,k_2)\} \\
& +\{(\nu,b,k_2)\rightarrow(\tau,d,k_4)),\ (\tau,d,k_4)\rightarrow(\lambda,c,k_3),\ (\lambda,c,k_3)\rightarrow(\nu,b,k_2)\}\ ,
\end{aligned}
\tag{21a}
$$

$$
\begin{aligned}
\Gamma_{(B)\mu\nu\lambda\tau}^{abcd}(k_1,\ k_2,\ k_3;q;\mu) \ = \ & -\frac{1}{2}g_c^{4-2q}\mu^{2q}[-6ig_c^2C_2(\mathbf{8})]^q\frac{1}{N_q}\sum_{i=0}^{q}\int\frac{d^4p}{(2\pi)^4} \\
& \times[f^{adr_1}f^{t_1s_1r_1}(g_{\mu\sigma_1}g_{\tau\rho_1}-g_{\mu\rho_1}g_{\tau\sigma_1})+f^{at_1r_1}f^{s_1dr_1}(g_{\mu\rho_1}g_{\tau\sigma_1}-g_{\mu\tau}g_{\rho_1\sigma_1}) \\
& +f^{as_1r_1}f^{dt_1r_1}(g_{\mu\tau}g_{\rho_1\sigma_1}-g_{\mu\sigma_1}g_{\tau\rho_1})][f^{bcr_2}f^{t_2s_2r_2}(g_{\nu\sigma_2}g_{\lambda\rho_2}-g_{\nu\rho_2}g_{\lambda\sigma_2}) \\
& +f^{bt_2r_2}f^{s_2cr_2}(g_{\nu\rho_2}g_{\lambda\sigma_2}-g_{\nu\lambda}g_{\rho_2\sigma_2})+f^{bs_2r_2}f^{ct_2r_2}(g_{\nu\lambda}g_{\rho_2\sigma_2}-g_{\nu\sigma_2}g_{\lambda\rho_2})] \\
& \times\delta^{s_1s_2}\delta^{t_1t_2}g^{\rho_1\rho_2}g^{\sigma_1\sigma_2}\Big(\frac{-i}{p^2}\Big)^{i+1}\Big(\frac{-i}{(p+k_2+k_3)^2}\Big)^{q-i+1} \\
& +\{(\tau,d,k_4)\leftrightarrow(\lambda,c,k_3)\}+\{(\tau,d,k_4)\leftrightarrow(\nu,b,k_2)\}\ ,
\end{aligned}
\tag{21b}
$$

$$
\begin{aligned}
\Gamma_{(C)\mu\nu\lambda\tau}^{abcd}(k_1,\ k_2,\ k_3;q;\mu) \ = \ & -ig_c^{4-2q}\mu^{2q}[-6ig_c^2C_2(\mathbf{8})]^q\frac{1}{N_q}\sum_{i=0}^{q}\sum_{j=0}^{q-i}\int\frac{d^4p}{(2\pi)^4} \\
& \times\delta^{s_1t_2}\delta^{s_2t_3}\delta^{s_3t_1}[f^{adr}f^{t_1s_1r}(g_{\mu\sigma_1}g_{\tau\rho_1}-g_{\mu\rho_1}g_{\tau\sigma_1}) \\
& +f^{at_1r}f^{s_1dr}(g_{\mu\rho_1}g_{\tau\sigma_1}-g_{\mu\tau}g_{\rho_1\sigma_1})+f^{as_1r}f^{dt_1r}(g_{\mu\tau}g_{\rho_1\sigma_1}-g_{\mu\sigma_1}g_{\tau\rho_1})] \\
& \times f^{bs_2t_2}f^{cs_3t_3}G_{\nu\rho_2\sigma_2}(k_2,-k_2-p,p)G_{\lambda\rho_3\sigma_3}(k_3,-k_2-k_3-p,k_2+p) \\
& \times g^{\rho_1\sigma_2}g^{\rho_2\sigma_3}g^{\rho_3\sigma_1}\Big(\frac{-i}{p^2}\Big)^{i+1}\Big(\frac{-i}{(p+k_2)^2}\Big)^{j+1}\Big(\frac{-i}{(p+k_2+k_3)^2}\Big)^{l+1} \\
& +\{(\tau,d,k_4)\leftrightarrow(\lambda,c,k_3)\}+\{(\tau,d,k_4)\leftrightarrow(\nu,b,k_2)\} \\
& +\{(\mu,a,k_1)\leftrightarrow(\lambda,c,k_3)\}+\{(\mu,a,k_1)\leftrightarrow(\nu,b,k_2)\} \\
& +\{(\mu,a,k_1)\leftrightarrow(\nu,b,k_2),\ (\tau,d,k_4)\leftrightarrow(\lambda,c,k_3)\}\ ,
\end{aligned}
\tag{21c}
$$

$$
\begin{aligned}
\Gamma_{(D)\mu\nu\lambda\tau}^{abcd}(k_1,\ k_2,\ k_3;q;\mu) \ = \ & -g_c^{4-2q}\mu^{2q}[-6ig_c^2C_2(\mathbf{8})]^qTr(\mathbf{F}^a\mathbf{F}^b\mathbf{F}^c\mathbf{F}^d)\frac{1}{N_q}\sum_{i=0}^{q}\sum_{j=0}^{q-i}\sum_{l=0}^{q-i-j} \\
& \times\int\frac{d^4p}{(2\pi)^4}p_\mu(p+k_2)_\nu(p+k_2+k_3)_\lambda(p-k_1)_\tau\Big(\frac{i}{p^2}\Big)^{i+1} \\
& \times\Big(\frac{i}{(p+k_2)^2}\Big)^{j+1}\Big(\frac{i}{(p+k_2+k_3)^2}\Big)^{l+1}\Big(\frac{i}{(p-k_1)^2}\Big)^{q-i-j-l+1} \\
& +\{(\nu,b,k_2)\rightarrow(\lambda,c,k_3),\ (\lambda,c,k_3)\rightarrow(\tau,d,k_4),\ (\tau,d,k_4)\rightarrow(\nu,b,k_2)\} \\
& +\{(\nu,b,k_2)\rightarrow(\tau,d,k_4)),\ (\tau,d,k_4)\rightarrow(\lambda,c,k_3),\ (\lambda,c,k_3)\rightarrow(\nu,b,k_2)\} \\
& +\{(\nu,b,k_2)\leftrightarrow(\lambda,c,k_3)\}+\{(\nu,b,k_2)\leftrightarrow(\tau,d,k_4)\}+\{(\lambda,c,k_3)\leftrightarrow(\tau,d,k_4)\},
\end{aligned}
\tag{21d}
$$

$$
\begin{aligned}
\Gamma_{(E)\mu\nu\lambda\tau}^{abcd}(k_1,\ k_2,\ k_3;q;\mu) \ = \ & -g_c^{4}\mu^{2q}(-i\lambda_q)^{2q}Tr(\mathbf{T}^a\mathbf{T}^b\mathbf{T}^c\mathbf{T}^d)\frac{1}{N_q}\sum_{i=0}^{2q}\sum_{j=0}^{2q-i}\sum_{l=0}^{2q-i-j} \\
& \times\int\frac{d^4p}{(2\pi)^4}Tr[\gamma_\mu\Big(\frac{i}{\not p-\not k_1-m}\Big)^{i+1}\gamma_\tau\Big(\frac{i}{\not p+\not k_2+\not k_3-m}\Big)^{j+1} \\
& \times\gamma_\lambda\Big(\frac{i}{\not p+\not k_2-m}\Big)^{l+1}\gamma_\nu\Big(\frac{i}{\not p-m}\Big)^{2q-i-j-l+1}] \\
& +\{(\nu,b,k_2)\rightarrow(\lambda,c,k_3),\ (\lambda,c,k_3)\rightarrow(\tau,d,k_4),\ (\tau,d,k_4)\rightarrow(\nu,b,k_2)\} \\
& +\{(\nu,b,k_2)\rightarrow(\tau,d,k_4)),\ (\tau,d,k_4)\rightarrow(\lambda,c,k_3),\ (\lambda,c,k_3)\rightarrow(\nu,b,k_2)\} \\
& +\{(\nu,b,k_2)\leftrightarrow(\lambda,c,k_3)\}+\{(\nu,b,k_2)\leftrightarrow(\tau,d,k_4)\}+\{(\lambda,c,k_3)\leftrightarrow(\tau,d,k_4)\}\ ,
\end{aligned}
\tag{21e}
$$

4. The integral expressions of the regularized diagram contributing to $\tilde{\Pi}^{ab}(p)$:

$$\tilde{\Pi}^{ab}(p;q;\mu) = -g_c^{2-2q}\mu^{2q}[-ig_c^2 C_2(\mathbf{8})]^q Tr(\mathbf{F}^a\mathbf{F}^b)\int \frac{d^4k}{(2\pi)^4} p\cdot k\Big(\frac{i}{k^2}\Big)^{q+1}\Big(\frac{-i}{(k-p)^2}\Big)\ , \tag{22}$$

5. The integral expressions of the regularized diagrams contributing to $\tilde{\Gamma}^{abc}_{\mu}(p',p)$:

$$\begin{aligned}\tilde{\Gamma}^{abc}_{(A)\mu}(p',p;q;\mu) &= ig_c^{3-2q}\mu^{2q}[-ig_c^2 C_2(\mathbf{8})]^q Tr(\mathbf{F}^a\mathbf{F}^b\mathbf{F}^c)\int \frac{d^4k}{(2\pi)^4} k^\sigma p'^\rho \\ &\quad \times G_{\mu\rho\sigma}(k_1-k_2,p-k_1,k_2-p)\Big(\frac{i}{k^2}\Big)^{q+1}\Big(\frac{-i}{(k-p')^2}\Big)\Big(\frac{-i}{(k-p)^2}\Big)\ ,\end{aligned} \tag{23a}$$

$$\begin{aligned}\tilde{\Gamma}^{abc}_{(B)\mu}(p',p;q;\mu) &= ig_c^{3-2q}\mu^{2q}[-6ig_c^2 C_2(\mathbf{8})]^q Tr(\mathbf{F}^a\mathbf{F}^b\mathbf{F}^c)\int \frac{d^4k}{(2\pi)^4}(k+p')_\mu(k+p)\cdot p' \\ &\quad \times\Big(\frac{-i}{k^2}\Big)^{q+1}\Big(\frac{i}{(k+p')^2}\Big)\Big(\frac{i}{(k+p)^2}\Big)\ ,\end{aligned} \tag{23b}$$

6. The integral expressions of the regularized diagram contributing to $\Sigma(p)$:

$$\begin{aligned}\Sigma(p;q;\mu) &= \mathbf{T}^a\mathbf{T}^a\Sigma^{QED}(p;q;\mu) \\ &= -g_c^{2-2q}\mu^{2q}[-6ig_c^2 C_2(\mathbf{8})]^q \mathbf{T}^a\mathbf{T}^a\int \frac{d^4k}{(2\pi)^4}(\gamma^\mu \frac{i}{\not{k}-m}\gamma_\mu)\Big(\frac{-i}{(k-p)^2}\Big)^{q+1},\end{aligned} \tag{24}$$

7. The integral expressions of the regularized diagrams contributing to $\Gamma^a_\mu(p',p)$:

$$\begin{aligned}\Gamma^{a}_{(A)\mu}(p',p;q;\mu) &= -g_c^{3-2q}\mu^{2q}[-6ig_c^2 C_2(\mathbf{8})]^q \mathbf{T}^s\mathbf{T}^t\frac{1}{N_q}\sum_{i=0}^{q}\int\frac{d^4k}{(2\pi)^4} \\ &\quad \times f^{ast}G_{\mu\rho\sigma}(p'-p,p-k,k-p') \\ &\quad \times(\gamma^\rho\frac{i}{\not{k}-m}\gamma^\sigma)\Big(\frac{-i}{(k-p)^2}\Big)^{i+1}\Big(\frac{-i}{(k-p')^2}\Big)^{q-i+1},\end{aligned} \tag{25a}$$

$$\begin{aligned}\Gamma^{a}_{(B)\mu}(p',p;q;\mu) &= \mathbf{T}^b\mathbf{T}^a\mathbf{T}^b\Gamma^{QED}(p',p;q;\mu) \\ &= -ig_c^{3-2q}\mu^{2q}[-6ig_c^2 C_2(\mathbf{8})]^q \mathbf{T}^b\mathbf{T}^a\mathbf{T}^b\int\frac{d^4k}{(2\pi)^4} \\ &\quad \times(\gamma^\rho\frac{i}{\not{k}+\not{p}'-m}\gamma_\mu\frac{1}{\not{k}+\not{p}-m}\gamma_\rho)\Big(\frac{-i}{k^2}\Big)^{q+1},\end{aligned} \tag{25b}$$

Acknowledgment

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Figure Captions

Figure 1 The one loop Feynman diagrams which are needed for calculation of the renormalization constant Z_3.

Figure 2 The one loop Feynman diagrams which are needed for calculation of the renormalization constant Z_1.

Figure 3 The one loop Feynman diagrams which are needed for calculation of the renormalization constant Z_4.

Figure 4 The one loop Feynman diagram which is needed for calculation of the renormalization constant $\tilde{Z}_3$.

Figure 5 The one loop Feynman diagrams which are needed for calculation of the renormalization constant $\tilde{Z}_1$.

Figure 6 The one loop Feynman diagram which is needed for calculation of the renormalization constant Z_3^F.

Figure 7 The one loop Feynman diagrams which are needed for calculation of the renormalization constant Z_1^F.

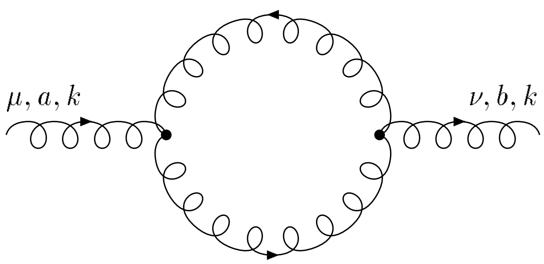

Fig.1a The gluon loop diagram.

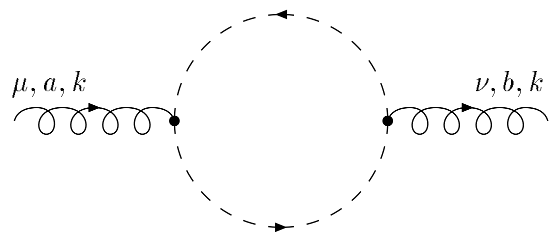

Fig.1b The ghost loop diagram.

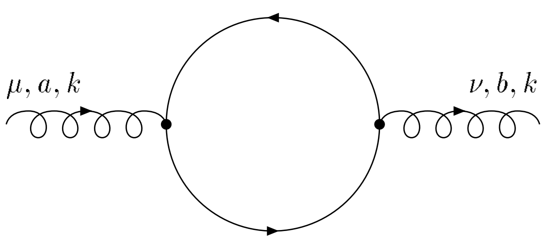

Fig.1c The quark loop diagram.

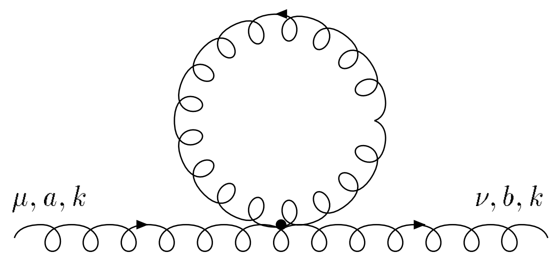

Fig.1d The gluon tadpole.

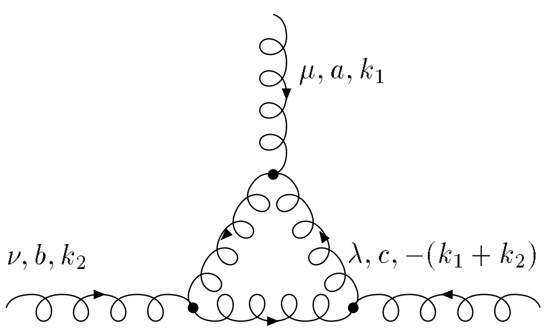

Fig.2a

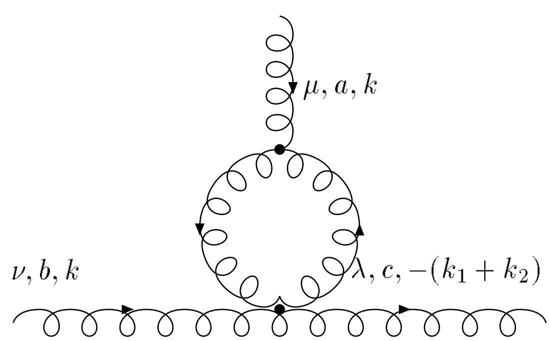

Fig.2b

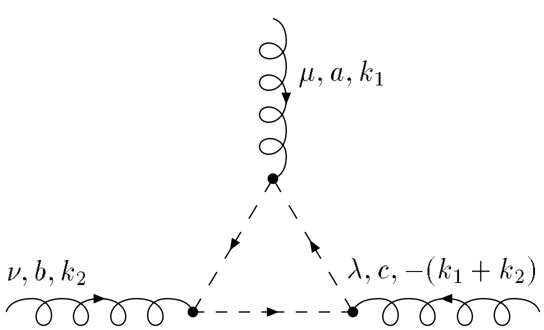

Fig.2c

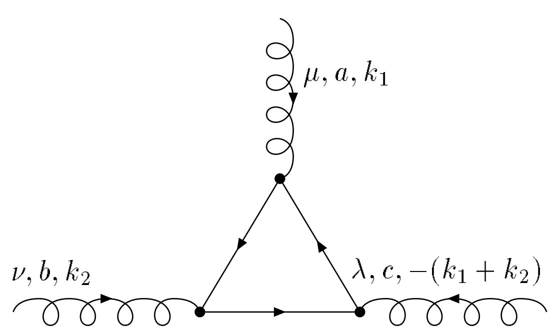

Fig.2d

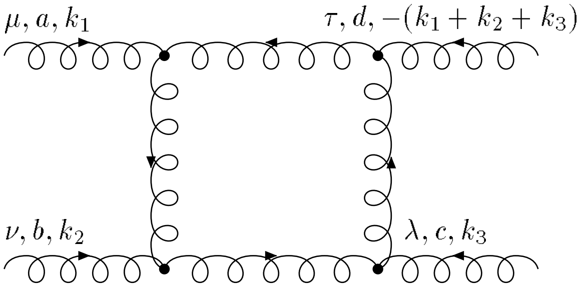

Fig.3a

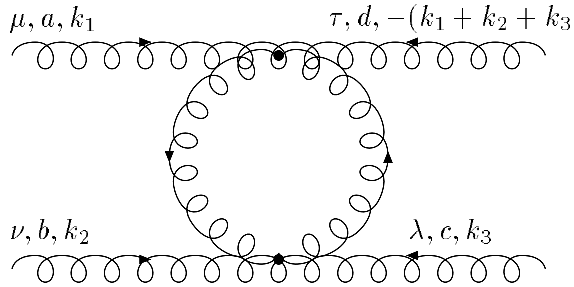

Fig.3b

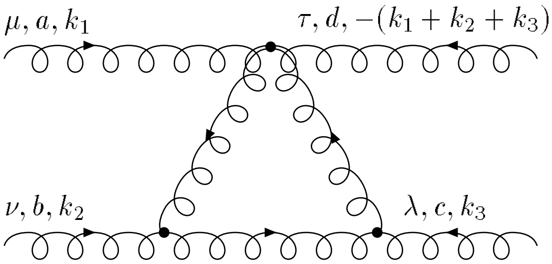

Fig.3c

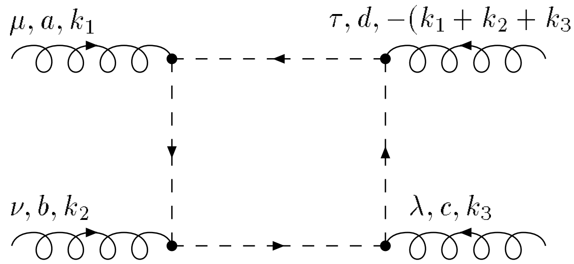

Fig.3d

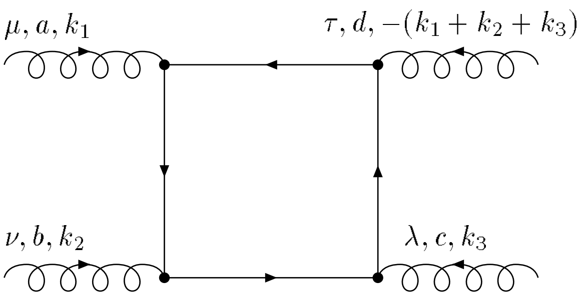

Fig.3e

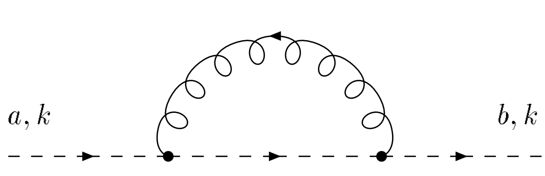

Fig.4

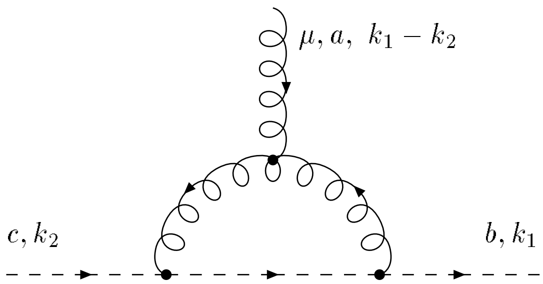

Fig.5a

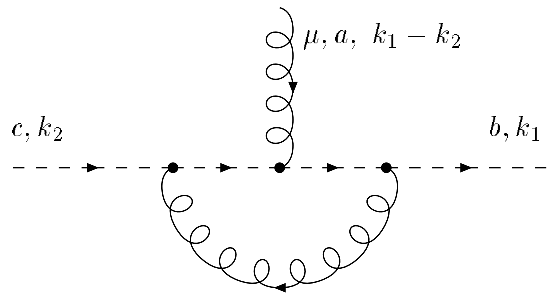

Fig.5b

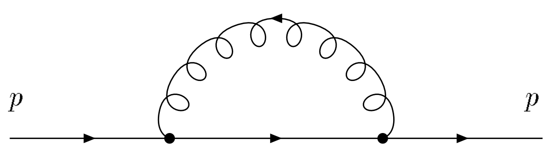

Fig.6

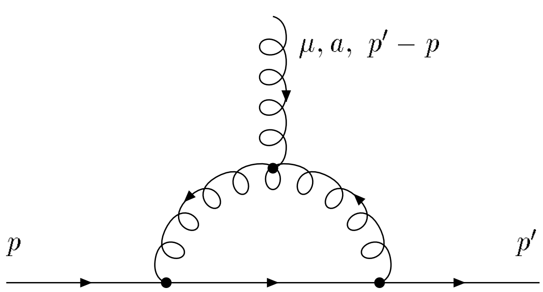

Fig.7a

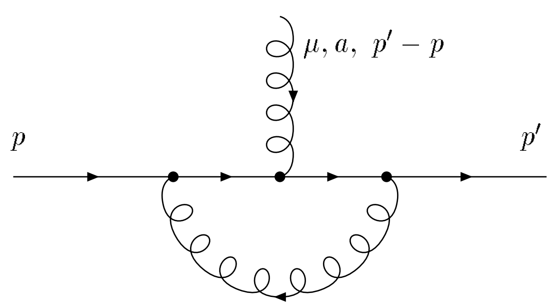

Fig.7b